Review Problems

1) Say everything you can about the following matrix.

$$\begin{bmatrix} 5 & 10 & 0 & 0 & 0 \\ 1 & 2 & 7 & 7 & 7 \\ 0 & 1 & 7 & 7 & 7 \\ 0 & 0 & 3 & 3 & 3 \end{bmatrix}$$

In particular, address the following incomplete list of interesting things we might like to say:

- What is its echelon form?
- What is its rank?
- What is the column space?
    - Its dimension?
    - A basis for it ?
- What is the row space?
    - Its dimension?
    - A basis for it ?
- What is the null space?
    - Its dimension?
    - A basis for it ?
- What is the associated linear operator T?
- What is the range of T?
    - Its dimension?
    - A basis for it ?
- What is the domain of T?
    - Its dimension?
    - A basis for it ?
- What is the codomain of T?
    - Its dimension?
    - A basis for it ?
- What is the kernel of T?
    - Its dimension?
    - A basis for it ?

2) Find 17 different bases for $\mathbb{R}^3$.

3) Find a linear operator $T: \mathbb{R}^4 \to \mathbb{R}^7$ with nontrivial kernel.

4) Given the matrix $[T] = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, find the T^{-1}, the inverse of the associated linear operator.

5) Find a linear operator T in which the inverse operator T^{-1} has trivial kernel.

6) Give an example of a dimension 2 space that is not $\mathbb{R}^2$.

7) Give an example of a dimension 34 space that is not $\mathbb{R}^{34}$.

8) A company produces x_1 phones and x_2 mp3 players. They have encoded their production information in to the following matrix equation using slack variables s_1 and s_2. Give one linear equation or inequality encoded here and describe what it means for this company.

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 20 & 50 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} 20{,}000 \\ 30{,}000 \\ 100{,}000 \end{bmatrix}$$

9) The set of vectors $\{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n\}$ is linearly dependent. Show that $\{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n, \vec{u}\}$ is also linearly dependent.